CHAPTER 6 Review

REFLECTING ON CHAPTER 6

- The work done in moving a mass (m) from a separation of r_1 to a separation of r_2 is given by
$$W = \frac{GMm}{r_1} - \frac{GMm}{r_2}$$
$$\text{or } W = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$
where M is the mass of the larger object.
- The work that must be done to move a mass m from a separation of r_1 to infinite separation is given by $W_{\text{to escape}} = \frac{GMm}{r_1}$.
- If the escape energy from a planet is to be provided through kinetic energy, the kinetic energy can be expressed as $\frac{1}{2}mv^2 = \frac{GMm}{r_p}$, where r_p is the radius of the planet. This leads to the escape speed being $v = \sqrt{\frac{2GM}{r_p}}$.
- The energies associated with circular orbits are $E_k = \frac{GMm}{2r}$, $E_g = -\frac{GMm}{r}$ and $E_{\text{total}} = -\frac{GMm}{2r}$.
- The gravitational potential energy of an object in the vicinity of a planet is negative, since work must be done on the object to remove it completely from the gravitational influence of the planet. The object is said to be in a gravitational potential energy well.
- For an object to be able to escape from the gravitational field of a planet, its total energy must be equal to or greater than zero. The extra energy that must be added to an object to bring its energy up to zero (thus allowing it to escape) is called the "binding energy" and is given by $E_{\text{binding}} = -E_{\text{total}}$.
- The magnitude of the thrust exerted by the gas being ejected from a rocket can be determined from the equation $F_{\text{thrust}} = \left(\frac{m_{\text{gas}}}{\Delta t}\right)\Delta v_{\text{gas}}$. This gas is known as "reaction mass."
- Gravitational assist is a technique by which a spacecraft gains kinetic energy from a planet around which it swings.

Knowledge/Understanding

1. **(a)** What must be true about the total orbital energy of any planet? Give reasons for your answer.
(b) What does it mean if some comets have total energies less than zero and others have total energies greater than zero?

2. Why does it take more energy to send a satellite into polar orbit (which follows Earth's longitudinal lines and passes over the North and South Poles) than into an equatorial orbit (which follows the equator eastward over Earth)?

3. For a satellite in circular orbit above Earth, state how the following properties depend on radius: **(a)** period; **(b)** kinetic energy; **(c)** speed.

Inquiry

4. Investigate the sizes of black holes by doing some simple escape velocity calculations. As postulated by Einstein's theory of general relativity, black holes are objects with such a strong gravitational field that their escape velocity exceeds the speed of light. Hence, nothing — not even light — can escape from a black hole. They are thought to be caused by massive stars that collapse in on themselves.
(a) What would be the escape velocity if, without losing any mass, Earth shrank to the following percentages of its present radius?
- $\frac{1}{100}$
- $\frac{1}{10000}$
- $\frac{1}{1 \times 10^6}$

(b) To how small a size would Earth have to collapse for its escape velocity to equal the speed of light?
(c) Although a full treatment of black holes requires general relativity, the radius (called the "Schwarzchild radius") can be calculated by using Newtonian theory: by setting $v_{\text{escape}} = c$ and solving for the radius. Suppose the core of a star 8.0 times more

massive than the Sun exhausted its fuel and collapsed. What size of black hole would form? (The Sun's mass is 1.99×10^{30} kg.)

(d) The centres of galaxies, including our own Milky Way, might contain black holes with masses of a million times the Sun's mass, or more. Calculate the size of a black hole with a mass of 1.99×10^{36} kg. Compare this to the size of the Sun's radius (6.96×10^8 m).

5. Consider a space shuttle in circular orbit around Earth. If the commander briefly fires a forward-pointing thruster so that the speed of the shuttle abruptly decreases, what would be the resulting effects on the kinetic energy, the total mechanical energy, the radius of the orbit, and the orbital period? Sketch the new orbit. Explain whether the new orbit will take the shuttle to the same point at which the thrusters were fired.

Communication

6. Explain how a satellite should be launched so that its orbit takes it over every point on Earth as Earth rotates.

7. What is the reason for choosing the zero of gravitational potential energy at infinity rather than, for example, at Earth's surface?

8. If the Sun shrank to the size of a black hole without losing any mass, what would happen to Earth's orbit?

9. Discuss whether you and your friends on the surface of Earth can be considered to be satellites orbiting Earth at a distance of 1.0 Earth radii.

10. **(a)** Explain whether the gravitational force of the Sun ever does work on a planet in a circular orbit.

(b) Does the gravitational force of the Sun ever do work on a comet in an elliptical orbit?

11. Suppose you launched a projectile in Halifax and it landed in Vancouver. If you assumed that the mass of Earth was concentrated at the centre, could you consider the projectile to be in temporary orbit around the centre of Earth? Explain your reasoning. Sketch the trajectory of the projectile over Earth. What would the complete orbit look like?

Making Connections

12. Investigate some of the details of rocket launches for interplanetary probes. What percentage of the probe's mass at lift-off is fuel? How much of the fuel is consumed at lift-off? If an expendable rocket is used to launch the probe, what options are currently available? How much does a launch cost? Summarize your findings in a report.

13. The United States recently announced plans to send an astronaut to Mars and return him or her safely to Earth. Such space travel is very costly compared to sending remote controlled probes, which can often collect as much information. Investigate the issues involved in interplanetary space travel and stage a debate in your class to examine them.

Problems for Understanding

14. Calculate the binding energy that a 50.0 kg classmate has while on the surface of Earth.

15. **(a)** What is the change in gravitational potential energy of a 6200 kg satellite that lifts off from Earth's surface into a circular orbit of altitude 2500 km?

(b) What percent error is introduced by assuming a constant value of g and calculating the change in gravitational potential energy from $mg\Delta h$?

16. The small ellipticity of Earth's orbit causes Earth's distance from the Sun to vary from 1.47×10^{11} m to 1.52×10^{11} m, with the average distance being 1.49×10^{11} m. The Sun's mass is 1.99×10^{30} kg. What is the change in Earth's gravitational potential energy as it moves from its smallest distance to its greatest distance from the Sun?

17. The mass of Mars is 0.107 times Earth's mass and its radius is 0.532 times Earth's radius. How does the escape velocity on Mars compare to the escape velocity on Earth?

18. The Sun's mass is 1.99×10^{30} kg and its radius is 6.96×10^8 m.

(a) Calculate the escape velocity from the Sun's surface.

(b) Calculate the escape velocity at the distance of Earth's orbit.

(c) Calculate the escape velocity at the distance of Pluto's orbit, 5.9×10^{12} m.

19. A rocket is launched vertically from Earth's surface with a velocity of 3.4 km/s. How high does it go **(a)** from Earth's centre and **(b)** from Earth's surface?

20. Calculate whether a major league baseball pitcher, who can pitch a fastball at 160 km/h (44 m/s), can pitch it right off of any of the following.

(a) Saturn's moon, Mimas, which has a mass of 3.8×10^{19} kg and a radius of 195 km.

(b) Jupiter's moon, Himalia, which has a mass of 9.5×10^{18} kg and a radius of 93 km.

(c) Mars' moon, Phobos, which has a mass of 1.1×10^{16} kg and a radius of 11 km.

(d) a neutron star, with 2.5 times the Sun's mass crammed into a radius of 8.0 km.

21. Consider an object at rest 1.00×10^2 Earth radii from Earth. With what speed will it hit the surface of Earth? Compare this to Earth's escape speed.

22. To exit from the solar system, the *Pioneer* spacecraft used a gravitational assist from Jupiter, which increased its kinetic energy at the expense of Jupiter's kinetic energy. If the spacecraft did not have this assist, how far out in the solar system would it travel? When it left Earth's vicinity, the spacecraft's speed, relative to the Sun, was 38 km/s.

23. A 650 kg satellite is to be placed into synchronous orbit around Earth.

(a) Calculate the gravitational potential energy of the satellite on Earth's surface.

(b) Calculate the total energy of the satellite while it is in its synchronous orbit with a radius of 4.22×10^7 m.

(c) What amount of work must be done on the satellite to raise it into synchronous orbit?

(d) Suppose that from its orbit you wanted to give the satellite enough energy to escape from Earth. How much energy would be required?

24. The atmosphere can exert a small air-drag force on satellites in low orbits and cause these orbits to decay.

(a) Despite an increased air-drag force as the orbit decays, the speed of the satellite increases. Show this by calculating the speed of a satellite when its altitude is 200 km (2.00×10^5 m) and when its altitude is 100 km (1.00×10^5 m).

(b) If the satellite's mass is 500 kg (5.00×10^2), show that the mechanical energy decreases, despite the increase in the satellite's kinetic energy.

25. One of the interesting things about a collapsed, compact object like a neutron star or a black hole is that, theoretically, a spacecraft could be sent close to it without suffering the effects of intense radiation. Consider a neutron star with a mass of 2.0 times the Sun's mass and a radius of 10.0 km. (**Note:** Think about the density that this implies!)

(a) Calculate the velocity of a spacecraft orbiting 500.0 km above the neutron star. (Note that at closer orbits, the effects of high gravity would need to be considered and the familiar formula used here would not apply.) What is this speed as a fraction of the speed of light?

(b) What is the spacecraft's period?

26. A rocket stands vertically on a launch pad and fires its engines. Gas is ejected at a rate of 1200 kg/s and the molecules have a speed of 40.0 km/s. What is the maximum weight the rocket can have if this thrust is to lift it slowly off the launch pad?

27. A 3.5×10^5 kg rocket stands vertically on a launch pad.

(a) Calculate the minimum thrust required to cause the rocket to rise from the launch pad.

(b) The rocket engines eject fuel at a rate of 28.0 kg/s. What is the velocity of the gas as it leaves the engine? Neglect the small mass change in the rocket due to the ejected fuel.